QUARTERLY PROGRESS REPORT 2

ELECTROMAGNETIC DEEP-PROBING (100-1000 KMS)

OF THE EARTH'S INTERIOR FROM ARTIFICIAL SATELLITES:

CONSTRAINTS ON THE REGIONAL EMPLACEMENT OF CRUSTAL RESOURCES

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Statement of Work

Objective

The objective of this investigation is to evaluate the applicability of electromagnetic deep-sounding experiments using natural sources in the magnetosphere by incorporating Magsat data with other geophysical data.

Approach

The investigator shall pursue the above objective through an analysis of Magsat satellite data, ground-based magnetic observations, appropriate reference field models, and other satellite data.

The objective will be pursued by seeking the optimal combination of observations which lead first to a global, and then to a regional, characterization of the conductivity of the Earth's upper mantle.

Tasks

The following tasks shall be performed by the investigator in fulfillment of the above objective:

a. Use data from Magsat satellite to constrain a long-period global "response function" for the average Earth at low latitudes over a period ranging from 6 hours to 27 days.

b. Synchronize the Magsat data with low-latitude ground-based observatory data to determine the vertical gradient of the respective magnetic field components. Use the vertical gradient of the appropriate components to independently ascertain the separation of external and internal field contributions.

c. Segregate the Magsat electromagnetic "response functions" according to the tectonic regime at the Earth's surface and evaluate systematic differences between regions having lateral scale sizes on the order of 1000 km or greater.

d. Theoretically evaluate problems of resolution and interpretation involving electromagnetic induction by temporally and spatially-varying magnetospheric sources in a rotating inhomogeneous Earth as observed at arbitrary points in space. Use these theoretical studies to constrain the interpretation of Magsat data as well as to propose further applications of satellite-based electromagnetic deep-sounding experiments.

e. Integrate the regional response functions with other geophysical data in order to constrain the joint interpretation of comprehensive physical models.

f. Prepare and submit to NASA periodic progress reports and a detailed final report documenting the results of this investigation.

CONSIDERATIONS IN NOISE-FREE ESTIMATES OF GLOBAL ELECTROMAGNETIC RESPONSE FUNCTIONS USING SATELLITE DATA

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Our preliminary goal in the MAGSAT project is to coordinate ground-based and satellite data sets. Toward this end we have developed, and are in the final stages of testing, a spherical harmonic analysis program which takes magnetic data in universal time from a set of arbitrarily spaced observatories and calculates a value for the instantaneous magnetic field at any point on the globe. The calculation is done as a least mean-squared value fit to a set of spherical harmonics up to any desired order n.

The program is also designed to accept as a set of input parameters the orbit position of a satellite and to coordinate it with ground-based magnetic data for a given time. The output is a predicted time series for the magnetic field on the earth's surface at the (r,θ) position directly under the hypothetical orbiting satellite for the duration of the time period of the input data set.

Using this program to "track" the surface magnetic field beneath the satellite will allow one to compute narrow-band averaged crosspowers between the spatially coordinated satellite and the ground-based data sets. These crosspowers can then be used to calculate field transfer coefficients with minimum noise distortion. As an example, we shall discuss the application of this technique to calculating the vector response function, W.

The following variables represent a narrow-band filtered frequency spectrum data set of the respective satellite magnetic component indicated (we assume the NEV coordinate system):

H_{SAT}: north component of satellite data

Z_{SAT}: vertical component of satellite data

Next, if we assume a P_1^0 external field, we can define:

$$H_c = H_{SAT}/\sin\theta$$
$$Z_c = Z_{SAT}/\cos\theta \quad (1)$$

to be the "latitude-compensated" components of the satellite data.

After Banks (1969), we define a vector response function W, where:

$$Z_c = WH_c \quad (2)$$

The usefulness of W centers on its independence of source field details. This result can be obtained from multi-layered spherical earth models, as long as the P_1^0 source field assumption is maintained.

Noting the form (2), we introduce the term W', a least-squares estimator of W; if we let V stand for the statistical variance of any estimation of W, say W^{est}, from W, then the following equation stands as our definition of W':

$$dV/dW' = \frac{d}{dW'}(\langle|Z_c - W'H_c|^2\rangle),$$
$$= 0, \quad (3)$$

where the angular brackets denote a frequency band average of the form:

$$\langle(Z_c-W'H_c)(Z_c-W'H_c)^*\rangle = \int_{\omega_0 \pm \Delta\omega/2} (Z_c(\omega)-W'H_c(\omega))(Z_c(\omega)-W'H_c(\omega))^* d\omega$$

where

ω_0: center frequency of the respective filter band,

$\Delta\omega$: the band width of the filter,

$Z_c(\omega)$, $H_c(\omega)$: elements of the respective spectra, Z_c and H_c.

In general we shall term "$\langle AB^*\rangle$ the "inner product" or "crossproduct" of A and B.

As a result of (3) we find that:

$$W' = \frac{\langle Z_c H_c^* \rangle}{\langle H_c H_c^* \rangle} \tag{4}$$

In a noise-free situation, W' would represent an exact calculation of W, up to the resolution of our instruments. As it is, however, our data will contain noise, such that H_c and Z_c may be expressed as

$$\begin{aligned} H_c &= H_c^{sig} + N_H \\ Z_c &= Z_c^{sig} + N_Z \end{aligned} \tag{5}$$

where N_H and N_Z are the noise spectra associated with the H and Z components, respectively.

We may evaluate how (4) will be affected by the noise. First, we see that the numerator, that is, the inner product of Z and H, will be essentially unaffected. We compute:

$$\begin{aligned} \langle Z_c H_c^* \rangle &= \langle (Z^{sig} + N_Z)((H_c^{sig})^* + N_H^*) \rangle \\ &= \langle Z_c^{sig}(H_c^{sig})^* + N_Z N_H^* + Z_c^{sig} N_H^* + N_Z (H_c^{sig})^* \rangle \\ &\approx \langle Z_c^{sig}(H_c^{sig})^* \rangle \end{aligned} \tag{6}$$

since $\langle N_Z N_H^* \rangle$ approaches zero because of the random and uncorrelated nature of noise sources. For the same reason, $\langle Z_c^{sig} N_H^* \rangle$ and $\langle N_Z (H_c^{sig})^* \rangle$ approach zero as well, so long as the selectivity of the band-limiting filter applied to the data is wide enough to allow the noise to average out.

Therefore, expression (6) supports our assertion that the numerator of (4) is unaffected by noise. The denominator, on the other hand, is decidedly affected:

$$\langle H_c H_c^* \rangle = \langle (H_c^{sig} + H_N)(H_c^{sig} + N_H)^* \rangle$$

$$= \langle |H_c^{sig}|^2 + |N_H|^2 \rangle \tag{7}$$

where the cross terms drop out according to the same reasoning employed in calculating (6).

The total effect of noise on our estimated parameter W' in (4), then, is a downward bias in magnitude (the denominator is real) relative to its noise-free value W. In other words, if we say

$$W = \frac{\langle Z_c^{sig} (H_c^{sig})^* \rangle}{\langle H_c^{sig} (H_c^{sig})^* \rangle} \tag{8a}$$

then

$$W' = \frac{\langle Z_c^{sig} (H_c^{sig})^* \rangle}{\langle \{H_c^{sig} (H_c^{sig})^*\} + \{N_H N_H^*\} \rangle} \tag{8b}$$

$$\leq W,$$

the equality holding in the limit as N_H approaches zero.

We note that only the noise in the H_c data affects the result. In effect, in formulating (2), we cast Z_c in the role of the "independent parameter", from which H_c could be obtained by way of the transfer function W. This was acceptable except that the noise in H, unaccountable for in our analytical transfer function, biased our final estimate.

Let us try a simple recasting of roles whereby H_c "becomes" independent and noise-free in the formulation. We define Y such that

$$H_c = YZ_c,$$

where in a noise-free situation,

$$1/Y = Z' = Z.$$

Performing a least-squares fit for Y as we did above for W', we obtain:

$$Y' = \frac{\langle H_c Z_c^* \rangle}{\langle Z_c Z_c^* \rangle} \tag{9a}$$

Then, performing a signal-noise separation as we did boding to (6) and (7), we find:

$$Y' = \frac{\langle H_c^{sig}(Z_c^{sig})^* \rangle}{\langle |Z_c^{sig}|^2 + |N_Z|^2 \rangle}$$

This result is analogous to (8b); the magnitude of Y' will be biased downward by the noise in Z_c, N_Z, as W' was biased downward by N_H. If we let

$$W'' = 1/Y' \tag{10}$$

W'' will then be biased <u>upward</u> by the N_Z. Hence we have:

$$W' < W < W'' \tag{11}$$

as a constraint on our hypothetical noise-free estimate, W.

A possible improvement would be to take the geometric mean of W' and W'', from (8b) and (9b) respectively, resulting in:

$$W^{AV} \equiv (W'W'')^{1/2} = \left(\frac{\langle |Z_c^{sig} + N_Z|^2 \rangle}{\langle |H_c^{sig} + N_H|^2 \rangle}\right)^{1/2} \cdot \left(\frac{\langle Z_c^{sig}(H_c^{sig})^* \rangle}{\langle H_c^{sig}(Z_c^{sig})^* \rangle}\right)^{1/2} \tag{12a}$$

If we express:

$$Z_c^{sig} = |Z_c^{sig}| e^{i\phi Z}$$

$$H_c^{sig} = |H_c^{sig}| e^{i\phi M}$$

then using this notation and the reasoning leading to (6), we have:

$$W^{AV} = \left(\frac{\langle |Z_c^{sig}|^2 + |N_Z|^2 \rangle}{\langle |H_c^{sig}|^2 + N_H| \rangle}\right)^{1/2} \quad e^{i(\phi_Z - \phi_H)}$$

Examination shows that W^{AV} is an excellent for the the magnitude of W if

$$\frac{|N_Z|^2}{|Z_c^{sig}|^2} \simeq \frac{|N_H|^2}{|H_c^{sig}|^2} \tag{13}$$

In addition, phase information is carried noise-free in W^{AV}, and is shown to be, equivalent to, the difference in the phase angles of the "hypothetical" noise-free values, "Z_c^{sig}" and "H_c^{sig}". Caution must be maintained, however, in using W_{AV} for data gathered at high colatitudes, where "Z_c^{sig}" is in general much smaller than "H_c^{sig}". In this case the approximate equality in (13) will break down. The necessary correction would involve reformulating "W_{AV}" in (12a) in the form of a weighted geometric mean, where the weight is a function of the appropriate coherencies.

The above discussion considered a method for extracting W from a data set from one site (a satellite in our instance). As we mentioned at the beginning of this discussion, the addition of a data set from another site, coordinated with the primary data set, will afford us a far more reliable method for extracting noise-free parameters and, in particular, for formulating W. Let us define

$$H_{ref} = H_{ref}^{raw}/\sin\theta$$

Analogous to our definitions in (1), H_{ref} represents the latitude-corrected narrow-band frequency spectrum for the horizontal, northern component of a ground-based data set, spatially and secularly coordinated with our satellite components H_c and Z_c.

Let us define a new field parameter as follows:

$$X = \frac{\langle Z_c H^*_{ref}\rangle}{\langle H_c H^*_{ref}\rangle} \qquad (15)$$

where we have taken (4) and replaced H_c^* with H^*_{ref}. We make the separation

$$H^*_{ref} = (H^{sig}_{ref})^* + N^*_{HR},$$

and assume that there is no statistical correlation between ground-based noise and satellite-based noise. It follows then from our arguments leading to (6) that

$$X = \frac{\langle Z_c^{sig}(H_{ref}^{sig})^* \rangle}{\langle H_c^{sig}(H_{ref}^{sig})^* \rangle} \tag{16}$$

Hence, (15) minimizes the effects of noise in an estimate of X. However, we must address the question of the physical interpretation of X; can it be equated with W? More specifically, what assumptions must we make about the relationship between H_{ref} and H_c so that "X = W" may be asserted? Another way of posing the question would be: in a noise-free situation, what constraints must be placed on H^*_{ref} so that its substitution for H^*_c into (4) would not disturb the identity?

Mathematically, the answer is simple: the frequency spectrum represented by H_{ref} must be a scalar multiple of the frequency spectrum represented by H_c for the substitution to be valid. In physical terms, this leads us to two basic constraints; one relates to the properties of the space between the ground and the satellite and the other relates to the conditioning of the data set itself.

For a spherically symmetric layered earth, assuming a P_1^0 source field, it can be shown that at or above the uppermost boundary,

$$H_c = [1 + \frac{R}{2}(r_1/r)^3]\frac{B_0}{\mu} \tag{17}$$

where r_1 is the radius of the uppermost boundary, r is the radius of the observer (it is presumed that $r \geq r_1$), B_0 is the magnitude of the external driving field, and R is a complex response function:

$$R = \frac{(i\omega\mu r_1 - 2Z')}{(i\omega\mu r_1 + Z')} \tag{18}$$

Z' is the surface impedance, calculated by iteration from the bottommost surface of the model.

The space between ground and satellite must have no sharp media boundaries and the conductivity throughout must be near-zero. Or else, reflection and/or

attenuation of the field between ground and satellite will occur in an uneven manner across the band of the frequency spectra of the magnetic components.

The data set conditioning must have a selectivity sufficiently narrow to insure that the response of the earth as a whole is essentially constant over the filter bandwidth. This last constraint represents a tradeoff with the assumptions made to reach (6), (7), (12) and (16), our various noise-minimized "W-estimators". We therefore must be aware of the special care necessitated in the choice of a proper selectivity, especially with long-period data.

We have presented these constraints under which "$X \cong W$" is a good approximation in a very qualitative manner. However, they are certainly basic problems that must be taken into account when applying any quantitative model to the data.

An attractive feature of the formulation proposed in (16) is the absence of a vertical ground-based component. This is an extremely useful property of this analysis as it is well known that surficial lateral inhomogeneities will introduce a distortion in the surface vertical component of far greater magnitude than the distortion introduced in the horizontal components.

In conclusion, we make two observations. First, as W is independent of source field details, its value as calculated from (16) should remain relatively constant as different segments of the 7-odd months of MAGSAT are processed via this formula. The size of the deviation of a set of "W's" thus calculated will provide us with a quasi-quantitative measure of the validity of the P_1^0 source field assumption which underlies the vector response function's source-independent character. This assumption can also be checked with ground-based data.

Finally, we note that although long-period data is of great use in our analysis as it suffers least from attenuation in the atmosphere or distortion by surficial inhomogeneities, it is also most difficult to extricate from the

data base. Satellite data close to the auroral zone must be discarded because of the large effect of the field-aligned currents have on the data. Therefore, we can have at most about 100 degrees of orbit time (about 23 minutes of data) for a continuous data set. We are at this time still looking for alternative methods of chaining data sets together which minimize possible effects of spatial aliasing.